Eur. Phys. J. C 32, 377–380 (2004)
Digital Object Identifier (DOI) 10.1140/epjc/s2003-01444-5

THE EUROPEAN
PHYSICAL JOURNAL C

Quantitative duality and neutral kaon interferometry

A. Bramon[1], G. Garbarino[2], B.C. Hiesmayr[1]

[1] Grup de Física Teòrica, Universitat Autònoma de Barcelona, 08193 Bellaterra, Spain
[2] Departament d'Estructura i Constituents de la Matèria, Universitat de Barcelona, 08028 Barcelona, Spain

Received: 17 July 2003 /
Published online: 3 December 2003 – © Springer-Verlag / Società Italiana di Fisica 2003

Abstract. A quantitative formulation of Bohr's complementarity principle and interferometric duality is discussed and applied to the neutral kaon system. Recent measurements by the CPLEAR Collaboration can be easily interpreted in terms of *neutral kaon interferometry*, illustrating and confirming those basic principles of quantum mechanics. The subject can be further investigated at the operational ϕ-factory Daphne.

1 Introduction

In his well-known Lectures on Physics, Feynman starts discussing the double-slit experiment as the most characteristic feature of quantum mechanics [1]: "In reality, it contains the *only* mystery." After this frequently quoted statement, an idealized but detailed analysis of the double-slit interference phenomenona is presented in terms of wave–particle duality, the rules for superposition of amplitudes and Bohr's complementarity principle. Somewhat later in the same Lectures, Feynman proceeds to discuss a particularly illustrative case: the neutral kaon system, for which he drastically concludes that [1]: "If there is any place where we have a chance to test the main principles of quantum mechanics in the purest way – does the superposition of amplitudes work or doesn't it? – this is it." In the present letter we reconsider these issues with a twofold purpose in mind. First, we show that a quantitative statement on duality – or "interferometric duality", as recently suggested by Englert [2] and reviewed in [3–5] –, which was originally proposed by Greenberger and Yasin [6], can be clearly illustrated using the K^0–$\bar{K}^0$ system. Secondly, this will then allow us to interpret two CPLEAR experiments on neutral kaons performed at CERN [7–9] as quantitative and elegant tests of duality and Bohr's complementarity in a new arena, which we would like to refer to as "neutral kaon interferometry".

2 Quantitative duality

The quantitative expression for Bohr's complementarity proposed by Greenberger and Yasin [6] is extremely simple. In today's most common notation [2–5], it reads

$$\mathcal{P}^2 + \mathcal{V}_0^2 \leq 1, \qquad (1)$$

where $\mathcal{V}_0$ is the "fringe visibility" which quantifies the sharpness or contrast of the interference pattern (a wave-like property), whereas $\mathcal{P}$ is the path "predictability" quantifying the a priori "which-way" knowledge one has of the path taken by the interfering object (its complementary particle-like property). In commonly used two-path interferometers – such as single-crystals in neutron interferometry [6, 10] or their Mach–Zehnder analogs in optical experiments, e.g. in [11] – one has to deal with two-level quantum states. They can be expressed as

$$|\Psi(\phi)\rangle = a|\psi_{\rm I}\rangle + b\,{\rm e}^{{\rm i}\phi}|\psi_{\rm II}\rangle, \qquad (2)$$

where a and b are positive, $a^2 + b^2 = 1$ and $|\psi_{\rm I}\rangle$ and $|\psi_{\rm II}\rangle$ represent the states corresponding to the two spatially separated interferometric paths, with (ideally) $\langle\psi_{\rm II}|\psi_{\rm I}\rangle = 0$ and a controllable relative phase ϕ. In the case of symmetric interferometers ($a = b = 1/\sqrt{2}$), the two paths are taken with the same probability, thus no a priori "which-way" knowledge is available, $\mathcal{P} = 0$, and maximal "fringe visibility" is expected, $\mathcal{V}_0 = 1$. In asymmetric cases ($a \neq b$), instead, the expression (1) becomes more interesting and quantifies the simultaneous wave and particle knowledge one can have for the interfering object according to Bohr's complementarity. Note that we are referring to a priori knowledge, which is fully contained in the preparation of the state (2). We thus exclude any possibility of knowledge-improving measurements (otherwise, another interesting inequality has been derived by Englert [2] and reviewed in [3–5]; its applicability to entangled neutral kaon systems has been recently proposed in [12]). If no knowledge-improving measurement is performed, the state (2) remains *pure* and fully coherent, and expression (1) is then verified with the *equal* sign.

The proof of this equality is quite simple, once the correct definitions and meanings for $\mathcal{V}_0$ and $\mathcal{P}$ are identified. The "fringe visibility", $\mathcal{V}_0$, is defined in the usual way as the coefficient $\mathcal{V}_0 \equiv (I_{\max} - I_{\min})/(I_{\max} + I_{\min})$ of the phase-dependent term in the expressions

$$I_\pm(\phi) = |\langle\psi_\pm|\Psi(\phi)\rangle|^2 = \frac{1}{2}[1 \pm \mathcal{V}_0 \cos\phi], \qquad (3)$$

which give two output intensities in two-channel interferometers in terms of ϕ. The amplitudes corresponding to paths $|\psi_I\rangle$ and $|\psi_{\mathrm{II}}\rangle$ are obtained by a first splitting of the initial beam and are later recombined before emerging from the two output ports. Most frequently and in (3), these two outputs are associated to measurements on the symmetric and antisymmetric basis states[1]

$$|\psi_\pm\rangle = \frac{1}{\sqrt{2}}[|\psi_{\mathrm{I}}\rangle \pm |\psi_{\mathrm{II}}\rangle]. \quad (4)$$

For the coherent superposition (2), rewritten in this new basis:

$$|\Psi(\phi)\rangle = \frac{1}{\sqrt{2}}\left(a + b\,\mathrm{e}^{\mathrm{i}\phi}\right)|\psi_+\rangle + \frac{1}{\sqrt{2}}\left(a - b\,\mathrm{e}^{\mathrm{i}\phi}\right)|\psi_-\rangle\,, \quad (5)$$

one easily obtains

$$\mathcal{V}_0 = 2ab. \quad (6)$$

The crucial concept of path "predictability", $\mathcal{P}$, turned out to be much harder to identify and was first introduced in [6]. Denoting by w_{I} and w_{II} the probabilities for taking either one of the two interferometric paths, $\mathcal{P}$ is defined as

$$\mathcal{P} \equiv |w_{\mathrm{I}} - w_{\mathrm{II}}| = \left|a^2 - b^2\right|, \quad (7)$$

where the final equality is specific of our pure state (2). From (6) and (7), the desired equality,

$$\mathcal{P}^2 + \mathcal{V}_0^2 = 1, \quad (8)$$

follows immediately. The use of the path "predictability" $\mathcal{P}$ introduced in [6] and defined in (7) – as opposed to other measures of "which-way" knowledge, as those introduced in [14] or in the theoretical-information approaches of [10,11,15] (for a critical discussion, see [16]) – has been crucial in order to derive (8). This equality can then be viewed as a modern and quantitative statement of Bohr's complementarity principle for pure states like (2).

3 Neutral kaon system

Pure states of neutral kaons have been copiously prepared at CPLEAR [7–9] using proton–antiproton annihilations at rest, $p\bar{p} \to K^-\pi^+K^0$ or $p\bar{p} \to K^+\pi^-\bar{K}^0$, where strangeness conservation in the strong interactions requires that a K^- has to be accompanied by a K^0 and a K^+ by a $\bar{K}^0$. In free space, the initially produced K^0 and $\bar{K}^0$ evolve in proper time τ according to the well-known expressions [17]

$$|K^0(\tau)\rangle = \frac{\sqrt{1+|\epsilon|^2}}{\sqrt{2}(1+\epsilon)}\left[\mathrm{e}^{-\mathrm{i}\lambda_{\mathrm{S}}\tau}|K_{\mathrm{S}}\rangle + \mathrm{e}^{-\mathrm{i}\lambda_{\mathrm{L}}\tau}|K_{\mathrm{L}}\rangle\right],$$

$$|\bar{K}^0(\tau)\rangle = \frac{\sqrt{1+|\epsilon|^2}}{\sqrt{2}(1-\epsilon)}\left[\mathrm{e}^{-\mathrm{i}\lambda_{\mathrm{S}}\tau}|K_{\mathrm{S}}\rangle - \mathrm{e}^{-\mathrm{i}\lambda_{\mathrm{L}}\tau}|K_{\mathrm{L}}\rangle\right],$$

[1] The photon experiment of [13] is an excellent example on the usefulness of the optical analogs of these basis states in a context related to ours

where ϵ is the (small) CP-violation parameter, $\lambda_{\mathrm{S,L}} \equiv m_{\mathrm{S,L}} - \frac{1}{2}\Gamma_{\mathrm{S,L}}$ and $m_{\mathrm{S,L}}$ and $\Gamma_{\mathrm{S,L}}$ are the masses and decay widths of the short- or long-lived states, K_{S} or K_{L}. By normalizing to kaons surviving up to time τ, the previous states can be more conveniently rewritten as

$$|K^0(\tau)\rangle = \frac{1}{\sqrt{1+\mathrm{e}^{-\Delta\Gamma\tau}}}\left[|K_{\mathrm{S}}\rangle + \mathrm{e}^{-\frac{1}{2}\Delta\Gamma\tau}\mathrm{e}^{-\mathrm{i}\Delta m\tau}|K_{\mathrm{L}}\rangle\right],$$
$$(9)$$
$$|\bar{K}^0(\tau)\rangle = \frac{1}{\sqrt{1+\mathrm{e}^{-\Delta\Gamma\tau}}}\left[|K_{\mathrm{S}}\rangle - \mathrm{e}^{-\frac{1}{2}\Delta\Gamma\tau}\mathrm{e}^{-\mathrm{i}\Delta m\tau}|K_{\mathrm{L}}\rangle\right],$$

where $\Delta m \equiv m_{\mathrm{L}} - m_{\mathrm{S}}$, $\Delta\Gamma \equiv \Gamma_{\mathrm{L}} - \Gamma_{\mathrm{S}}$ and, even if one strictly has $\langle K_{\mathrm{S}}|K_{\mathrm{L}}\rangle = (\epsilon+\epsilon^*)/(1+|\epsilon|^2) \simeq 3.2\times10^{-3}$, we neglect this small CP-violation effect by taking K_{S} and K_{L} as orthogonal states, $\langle K_{\mathrm{S}}|K_{\mathrm{L}}\rangle = 0$.

The situation then mimics perfectly that of the two-path interferometers previously discussed and admits the same formalism. The approximation $\langle K_{\mathrm{S}}|K_{\mathrm{L}}\rangle = 0$ just introduced corresponds to the (similarly approximate) two-path orthogonality $\langle\psi_{\mathrm{I}}|\psi_{\mathrm{II}}\rangle = 0$, and the states (9) are particular cases of the state (2) with

$$a = \frac{1}{\sqrt{1+\mathrm{e}^{-\Delta\Gamma\tau}}}, \quad b = \frac{1}{\sqrt{1+\mathrm{e}^{+\Delta\Gamma\tau}}},$$
$$\phi = -\Delta m\,\tau \;\text{ or }\; \pi - \Delta m\,\tau.$$

Similarly, within our CP-conserving approximation, the strictly orthogonal $\{K^0, \bar{K}^0\}$ basis is related to the $\{K_{\mathrm{S}}, K_{\mathrm{L}}\}$ basis by

$$|K^0\rangle = \frac{1}{\sqrt{2}}[|K_{\mathrm{S}}\rangle + |K_{\mathrm{L}}\rangle], \quad |\bar{K}^0\rangle = \frac{1}{\sqrt{2}}[|K_{\mathrm{S}}\rangle - |K_{\mathrm{L}}\rangle], \quad (10)$$

in close analogy with (4). In this $\{K^0, \bar{K}^0\}$ basis one has

$$|K^0(\tau)\rangle = \quad (11)$$
$$\frac{1}{\sqrt{2}}\left[\frac{1+\mathrm{e}^{-\frac{1}{2}\Delta\Gamma\tau}\mathrm{e}^{-\mathrm{i}\Delta m\tau}}{\sqrt{1+\mathrm{e}^{-\Delta\Gamma\tau}}}|K^0\rangle + \frac{1-\mathrm{e}^{-\frac{1}{2}\Delta\Gamma\tau}\mathrm{e}^{-\mathrm{i}\Delta m\tau}}{\sqrt{1+\mathrm{e}^{-\Delta\Gamma\tau}}}|\bar{K}^0\rangle\right],$$
$$|\bar{K}^0(\tau)\rangle = \quad (12)$$
$$\frac{1}{\sqrt{2}}\left[\frac{1-\mathrm{e}^{-\frac{1}{2}\Delta\Gamma\tau}\mathrm{e}^{-\mathrm{i}\Delta m\tau}}{\sqrt{1+\mathrm{e}^{-\Delta\Gamma\tau}}}|K^0\rangle + \frac{1+\mathrm{e}^{-\frac{1}{2}\Delta\Gamma\tau}\mathrm{e}^{-\mathrm{i}\Delta m\tau}}{\sqrt{1+\mathrm{e}^{-\Delta\Gamma\tau}}}|\bar{K}^0\rangle\right],$$

thus mimicking (5). For the "fringe visibility" and the a priori "predictability" one now obtains the *time-dependent* expressions:

$$\mathcal{V}_0(\tau) = \frac{2}{\sqrt{2+\mathrm{e}^{-\Delta\Gamma\tau}+\mathrm{e}^{+\Delta\Gamma\tau}}} = \frac{1}{\cosh\left(\frac{1}{2}\Delta\Gamma\tau\right)}, \quad (13)$$

$$\mathcal{P}(\tau) = \left|\frac{1}{1+\mathrm{e}^{-\Delta\Gamma\tau}} - \frac{1}{1+\mathrm{e}^{+\Delta\Gamma\tau}}\right|$$
$$= \tanh\left|\frac{1}{2}\Delta\Gamma\tau\right|, \quad (14)$$

which obviously verify for all values of τ the time-dependent version of (8):

$$\mathcal{P}(\tau)^2 + \mathcal{V}_0(\tau)^2 = 1, \tag{15}$$

as expected for pure states such as $|K^0(\tau)\rangle$ and $|\bar{K}^0(\tau)\rangle$.

The physical interpretation of these kaonic results seems unique and quite obvious. As soon as a K^0 or a $\bar{K}^0$ is produced by strangeness-conserving strong interactions, it starts propagating in free space in the coherent superposition (10) of the $|K_\mathrm{S}\rangle$ and $|K_\mathrm{L}\rangle$ components. These two components, which evolve in time without oscillating into each other, decrease exponentially at remarkably different decay rates ($\Gamma_\mathrm{S} \simeq 579\,\Gamma_\mathrm{L}$). The $|K_\mathrm{S}\rangle$ and $|K_\mathrm{L}\rangle$ states are thus the analogs of the two separated paths $|\psi_\mathrm{I}\rangle$ and $|\psi_\mathrm{II}\rangle$ (associated with particle-like behaviour) in usual interferometers, with $\langle K_\mathrm{L}|K_\mathrm{S}\rangle \simeq \langle\psi_\mathrm{II}|\psi_\mathrm{I}\rangle \simeq 0$. Much in the same way as the probabilities for taking each one of the two paths are not equal in asymmetric interferometers, the probabilities for $|K_\mathrm{S}\rangle$ or $|K_\mathrm{L}\rangle$ propagation are similarly different except at $\tau = 0$. Indeed, for undecayed kaons surviving up to $\tau > 0$ one knows that (slowly decaying) $|K_\mathrm{L}\rangle$ propagation is more likely than its (much faster decaying) alternative $|K_\mathrm{S}\rangle$. One has an a priori knowledge or "predictability" on the actual propagation path, which is merely a consequence of knowing the state one is dealing with, as in the case of asymmetric interferometers.

The role of "fringe visibility" in ordinary interferometry (associated to the complementary wave-like behaviour) is played by the well-known phenomenon of strangeness oscillations in the neutral kaon case. As previously mentioned, in ordinary interferometers (such as those of [10,11]) the two amplitudes corresponding to the $|\psi_\mathrm{I}\rangle$ and $|\psi_\mathrm{II}\rangle$ paths have to be recombined before emerging from the two output ports, where a projective measurement in the $\{\psi_+, \psi_-\}$ basis is performed. In the neutral kaon case one takes advantage of the strong analogy between the bases (4) and (10), and of the fact that the amplitudes for $|K_\mathrm{S}\rangle$ and $|K_\mathrm{L}\rangle$ propagation are automatically recombined if a projective measurement in the $\{K^0, \bar{K}^0\}$ basis, corresponding to strangeness $S = \pm 1$, is performed. There are two independent and time-honoured methods for performing these measurements (a short historical review may be found in [7]) and both have been successfully used by the CPLEAR collaboration, as we now proceed to discuss.

4 CPLEAR experiments

In a recent CPLEAR experiment [7], K^0–$\bar{K}^0$ oscillations have been observed via strangeness measurements monitored by kaon–nucleon strong interactions. The previously mentioned proton–antiproton annihilation processes, $p\bar{p} \to K^-\pi^+K^0$ or $p\bar{p} \to K^+\pi^-\bar{K}^0$, were used to produce initial $|K^0\rangle$ or $|\bar{K}^0\rangle$ states, which were allowed to propagate in free space. The strangeness of the states $|K^0(\tau)\rangle$ and $|\bar{K}^0(\tau)\rangle$, (11) and (12), was subsequently measured at different proper times τ. This was achieved by inserting a 2.5 cm thick carbon absorber which allowed one to detect neutral kaon interactions with nucleons in the time interval 1.3–5.3 τ_S. The number of K^0 and $\bar{K}^0$ interacting with the absorber's bound nucleons via $K^0 + p \to K^+ + n$ (thus projecting on the $|K^0\rangle$ state) or, alternatively, via $\bar{K}^0 + n \to K^- + p$, $\bar{K}^0 + n \to \pi^0 + \Lambda(\to \pi^- + p)$ (projecting on $|\bar{K}^0\rangle$) were carefully recorded together with the vertex position or interaction time τ. An asymmetry parameter, $A^\mathrm{strong}_{\Delta m}(\tau)$, conveniently minimizing some experimental uncertainties, was then defined (see (5) in [7]) and used to extract a value for Δm – fully compatible with other measurements– by fitting the time dependence of these data.

These findings, however, admit an independent and immediate interpretation in terms of the "kaon interferometry" we are discussing. Indeed, the measured asymmetry parameter, $A^\mathrm{strong}_{\Delta m}(\tau)$, can be easily rewritten as

$$A^\mathrm{strong}_{\Delta m}(\tau) = \frac{2\mathcal{V}_0(\tau)\cos(\Delta m\tau)}{1 + \mathcal{V}_0^2(\tau)\cos^2(\Delta m\tau)},$$

with $\mathcal{V}_0(\tau) = 1/\cosh(\Delta\Gamma\tau/2)$, as given in (13). In other words, the CPLEAR data [7] can be viewed as the successful measurement of the "fringe visibility" of strangeness oscillations according to our discussion. By jointly considering these $\mathcal{V}_0(\tau)$ measurements with the complementary "which-path" information $\mathcal{P}(\tau)$ – which is simply given by inserting the observed interaction time τ and the well-known values of Γ_S and Γ_L in (14)–, these CPLEAR results are seen to fulfil the statement for quantitative duality, (15), for the whole range of τ-values 1.3–5.3 τ_S.

In another CPLEAR experiment [8], equivalent results were obtained and the whole picture is confirmed. Here strangeness oscillations were observed by detecting semileptonic neutral kaon decays. According to the well-tested $\Delta S = \Delta Q$ rule, the $K^0 \to \mathrm{e}^-\pi^+\bar{\nu}$ and $\bar{K}^0 \to \mathrm{e}^+\pi^-\nu$ decays are forbidden and, therefore, whenever observed, these semileptonic final states have to be necessarily associated with weak decays from $\bar{K}^0$ and K^0 states, respectively. This represents an independent method of strangeness measurement and a new asymmetry parameter, $A^\mathrm{weak}_{\Delta m}(\tau)$ ((2) in [8]), can be defined in such a way that (3) becomes $I_\pm(\tau) = [1 \pm A^\mathrm{weak}_{\Delta m}(\tau)]/2$. The $A^\mathrm{weak}_{\Delta m}(\tau)$ measurement shows again a characteristic oscillatory τ-dependence and has been fitted to extract an independent value for Δm, in agreement with the present world average and even having the same accuracy. Alternatively, one can reinterpret these results as before. Indeed, assuming no violation of the $\Delta S = \Delta Q$ rule, one easily finds

$$A^\mathrm{weak}_{\Delta m}(\tau) = \mathcal{V}_0(\tau)\cos(\Delta m\tau) = \frac{\cos(\Delta m\tau)}{\cosh\left(\frac{1}{2}\Delta\Gamma\tau\right)},$$

as required by (13). Again, using these CPLEAR data to extract values of $\mathcal{V}_0(\tau)$ and $\mathcal{P}(\tau)$ would confirm the validity of (15) from $\tau \simeq 1.5\,\tau_\mathrm{S}$ to $\tau \simeq 20\,\tau_\mathrm{S}$.

Prompt strangeness measurement events at $\tau << \tau_\mathrm{S}$, for which one predicts $\mathcal{V}_0(\tau << \tau_\mathrm{S}) \simeq 1$ and $\mathcal{P}(\tau << \tau_\mathrm{S}) \simeq 0$, were not performed by the CPLEAR collaboration. Note, however, that both sets of CPLEAR data include measurements around $\tau = 1.8\,\tau_\mathrm{S}$ showing a contrasted oscillatory behaviour $[\mathcal{V}_0(\tau = 1.8\,\tau_\mathrm{S}) \simeq 0.7]$ because the available information on which component, K_S or K_L, is actually propagating is still incomplete [$\mathcal{P}(\tau = 1.8\,\tau_S) \simeq 0.7$] and cannot be in any way increased. Indeed, these observed neutral kaons have been converted into another hadron or a semileptonic final state once their strangeness has been measured as in [7] and [8], respectively. Conversely, kaons surviving up to $\tau \simeq 5\,\tau_S$ (or more) are known to propagate as K_Ls almost for sure [$\mathcal{P}(\tau \gtrsim 5\,\tau_S) \simeq 1$]. Indeed, the probability that a K_S survives up to $5\,\tau_S$ reduces to a few per thousand, which is of the same order as the CP-violation effects we are systematically neglecting, and thus beyond the accuracy of our present approximate treatment. Consequently, the contrast of strangeness oscillations is seen to disappear when approaching these larger τ-values.

5 Conclusions

In our view, the previous discussion exemplifies in a clear way the concept of quantitative duality, for which a general formulation has been recently proposed by Englert in the following terms [2]: "*Duality* –The observation of an interference pattern and the acquisition of which-way information are mutually exclusive."

As anticipated by Feynman's quotations in the Introduction, the extension of these ideas to "neutral kaon interferometry" is extremely simple and illustrative. The transition from maximal "fringe visibility" at $\tau = 0$ – with no information on which component actually propagates – to the opposite extreme case, $\tau >> \tau_S$, allows one to cover the intermediate stages automatically by measuring strangeness at intermediate times τ. This leads to simple τ-dependent expressions for $\mathcal{V}_0(\tau)$ and $\mathcal{P}(\tau)$ in terms of $\Delta\Gamma\,\tau$, whereas the oscillatory term itself is exclusively driven by $\Delta m\,\tau$. In this sense, neutral kaon interference experiments in free space are universally governed by these two well-determined parameters alone: $\Delta\Gamma$ and Δm. Note however that the time dependence of $\mathcal{V}_0(\tau)$ and $\mathcal{P}(\tau)$, which cannot be avoided for unstable kaons, is specific of our context[2]. It allows for further and independent tests of basic principles in terms of (15)[3].

To the best of our knowledge, only the neutron experiments of [10] and the photon experiment of [11] have attempted to test these ideas. In both cases, an older version of the interferometric duality – which was originally introduced in [15] and, according to [16], involves a less satisfactory measure of the "which-way" information (the information-theoretical "lack of knowledge" rather than "predictability") – was used.

In conclusion, we have shown that two CPLEAR experiments, which admit a transparent interpretation in terms of (15), are fully consistent with this equality. Further experiments at the operational ϕ-factory Daphne [19], which copiously produces neutral kaons via strong $\phi \to K^0\bar{K}^0$ decays, are going to be of interest.

[2] Note also that, thanks to its similarities with the $K^0\bar{K}^0$ system, the same discussion as in Sect. 3 applies to the $B^0\bar{B}^0$ system. The main difference is due to the values of the B-meson parameters, for which experimentally one only knows that $|\Delta\Gamma_B| << \Delta m_B$. The number of oscillations that one can observe is thus much larger than in the kaon case, where $|\Delta\Gamma| \simeq 2\,\Delta m$

[3] Most frequently, experiments are expected to test (8) [rather than (15)] where $\mathcal{V}_0$ and $\mathcal{P}$ are constants in a given experimental set-up. A curious exception is the optical interferometer proposed in [18], where the "fringe visibility" is found to have the same functional dependence (in space) as (13) (in time)

Acknowledgements. This work has been partly supported by EURIDICE HPRN-CT-2002-00311, BFM-2002-02588, and INFN. Discussions with M. Nowakowski are acknowledged.

References

1. R.P. Feynman, R.B. Leighton, M. Sands, The Feynman Lectures on Physics (Addison-Wesley, Reading, MA, 1965), Vol. III
2. B.-G. Englert, Phys. Rev. Lett. **77**, 2154 (1996)
3. B.-G. Englert, Z. Naturforsch. **54**a, 11 (1999)
4. B.-G. Englert, J.A. Bergou, Opt. Commun. **179**, 337 (2000)
5. S. Dürr, G. Rempe, Opt. Commun. **179**, 323 (2000)
6. D. Greenberger, A. Yasin, Phys. Lett. A **128**, 391 (1988)
7. A. Angelopoulos et al., Phys. Lett. B **503**, 49 (2001)
8. A. Angelopoulos et al., Phys. Lett. B **444**, 38 (1998)
9. For a comprehensive review of CPLEAR results, see A. Angelopoulos et al., Phys. Rept. **374**, 165 (2003)
10. H. Rauch, J. Summhammer, Phys. Lett. A **104**, 44 (1984); J. Summhammer, H. Rauch, D. Tuppinger, Phys. Rev. A **36**, 4447 (1987)
11. P. Mittelstaedt, A. Prieur, R. Schieder, Found. Phys. **17**, 891 (1987)
12. A. Bramon, G. Garbarino, B.C. Hiesmayr, quant-ph/0306114, Phys. Rev. Lett. (in press)
13. T.J. Herzog, P.G. Kwiat, H. Weinfurter, A. Zeilinger, Phys. Rev. Lett. **75**, 3034 (1995)
14. L. Mandel, Opt. Lett. **16**, 1882 (1991)
15. W.K. Wootters, W.K. Zurek, Phys. Rev. D **19**, 473 (1979)
16. G. Jaeger, A. Shimony, L. Vaidman, Phys. Rev. A **51**, 54 (1995)
17. P.K. Kabir, The CP puzzle (Academic Press, London 1968)
18. L.S. Bartell, Phys. Rev. D **21**, 1698 (1980)
19. For a survey on the physics at a Φ-factory see The Second DaΦne Physics Handbook, edited by L. Maiani, G. Pancheri, N. Paver (INFN, Laboratori Nazionali di Frascati 1995)